Notes on Converting Repeating Decimals to Fractions

Example 1: Convert $0.\overline{5}$ to a fraction.

Let $x = 0.\overline{5}$
Then $10x = 5.\overline{5}$
Subtract: $10x = 5.\overline{5}$
\[-(x = 0.\overline{5})\]
\[9x = 5\]

Solve for $x$: $\frac{9x}{9} = \frac{5}{9}$

Dividing both sides of the equation by 9

\[x = \frac{5}{9}\]

Step 1 Set $x =$ repeating decimal (Remove whole number. Place in answer, for later.)

Step 2 Get repeater next to the decimal by multiplying both sides of the equation by a multiple of 10.

Step 3 Find a second equation with the same repeater next to the decimal. Again, do this by multiplying both sides of the equation by another multiple of 10.

Step 4 Subtract Step 2 from Step 3

Step 5 Solve for $x$.

Step 6 Simplify

(turn over for another example)
Example 2: Convert $2.\overline{135}$ to a fraction.

(1) Let $x = 0.\overline{135}$ (Remove 2 for later; it is the whole number in the answer.)

(2) Then $10x = 1.\overline{35}$ Both sides of the equation were multiplied by 10 so that the repeating part of the number is immediately next to the decimal.

(3) Then $1000x = 135.\overline{35}$ Here, both sides of the (original) equation were multiplied by 1000 so that the SAME repeating part of the number is immediately next to the decimal.

(4) Subtract: $1000x = 135.\overline{35}$ Subtract step 2 from step 3. Note how the repeating decimal drops out of the equation and the coefficients (leading numbers) are both whole numbers. Beautiful!

\[
\begin{align*}
990x &= 134 \\
\end{align*}
\]

(5) Solve for $x$: $\frac{990x}{990} = \frac{134}{990}$ Divide both sides of the equation by the same value—in this case, 990.

(6) Simplify: $x = \frac{134}{990} = \frac{68}{495}$ ...using GIANT ONE.

Final answer: $2.\overline{135} = 2\frac{68}{495}$ Remember that your answer includes the whole number from the beginning of the problem. Also, always check to see if your answer makes sense!