Name_____

Notes on Converting Repeating Decimals to Fractions

<u>Example 1:</u> Convert $0.\overline{5}$ to a fraction.

Let $x = 0.\overline{5}$

Then $10x = 5.\overline{5}$

Subtract: $10x = 5.\overline{5}$

$\quad\quad\quad \underline{-(x = 0.\overline{5})}$ $\quad$ | Note: *x = 1x* |

$\quad\quad\quad\quad 9x = 5$

Solve for x: $\dfrac{9x}{9} = \dfrac{5}{9}$ $\quad$ | Dividing both sides of the equation by 9 |

$\quad\quad\quad\quad\quad x = \dfrac{5}{9}$

<u>Step 1</u> Set x = repeating decimal (Remove whole number. Place in answer, for later.)

<u>Step 2</u> **Get repeater next to the decimal** by multiplying both sides of the equation by a multiple of 10.

<u>Step 3</u> Find a second equation with the **<u>same repeater</u>** <u>next to the decimal</u>. Again, do this by multiplying both sides of the equation by another multiple of 10.

<u>Step 4</u> **Subtract Step 2 from Step 3**

<u>Step 5</u> Solve for x.

<u>Step 6</u> Simplify

<u>Example 2:</u> Convert $2.1\overline{35}$ to a fraction.

(1) Let $x = 0.1\overline{35}$ (Remove 2 for later; it is the whole number in the answer.)

(2) Then $10x = 1.\overline{35}$ | Both sides of the equation were multiplied by 10 so that the repeating part of the number is immediately next to the decimal.

(3) Then $1000x = 135.\overline{35}$ | Here, both sides of the (original) equation were multiplied by 1000 so that the SAME repeating part of the number is immediately next to the decimal.

(4) Subtract: $1000x = 135.\overline{35}$
$\quad\quad\quad\; - \;\; 10x = 1.\overline{35}$
$\quad\quad\quad\quad\; 990x = 134$

Subtract step 2 from step 3. Note how the repeating decimal drops out of the equation and the coefficients (leading numbers) are both whole numbers. Beautiful!

(5) Solve for x: $\dfrac{990x}{990} = \dfrac{134}{990}$ | Divide both sides of the equation by the same value--in this case, 990.

(6) Simplify: $x = \dfrac{134}{990} = \dfrac{68}{495}$ | ...using GIANT ONE.

Final answer: $2.1\overline{35} = 2\dfrac{68}{495}$ | Remember that your answer includes the whole number from the beginning of the problem. Also, always check to see if your answer makes sense!