Rules of Exponents Name____________________________________

Example: 5^2

base: 5 exponent: 2 power: 2nd

read: five squared or five to the second power

Exponents are used in many algebra problems, so it's important that you understand the rules for working with exponents. Let's go over each rule in detail, and see some examples.

Zero Rule

According to the "zero rule," any nonzero number raised to the power of zero equals 1.

$$x^0 = 1 \quad \text{(as long as } x \neq 0\text{)}$$

Rules of 1

There are two simple "rules of 1" to remember.

First, any number raised to the power of "one" equals itself. This makes sense, because the power shows how many times the base is multiplied by itself. If it's only multiplied one time, then it's logical that it equals itself.

Secondly, one raised to any power is one. This, too, is logical, because one times one times one, as many times as you multiply it, is always equal to one.

$$x^1 = x$$

$$3^1 = 3$$

$$1^m = 1$$

$$1^4 = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

Product Rule

The exponent "product rule" tells us that, when multiplying two powers that have the <u>same</u> <u>base</u>, you can add the exponents. In this example, you can see how it works. Adding the exponents is just a short cut!

$$x^m \cdot x^n = x^{m+n}$$

$$4^2 \cdot 4^3 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^{2+3} = 4^5$$

Quotient Rule

The quotient rule tells us that we can divide two powers with the same base by subtracting the exponents. You can see why this works if you study the example shown.

$$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n} \qquad \text{(Note: } x \neq 0\text{.)}$$

$$4^5 \div 4^2 = \frac{4^5}{4^2} = \frac{\cancel{4} \cdot \cancel{4} \cdot 4 \cdot 4 \cdot 4}{\cancel{4} \cdot \cancel{4}} = 4^{5-2} = 4^3$$

(Note: Giant One shown using / marks. Numbers are not eliminated; they are simplified to 1.)

Negative Exponents

The last rule in this lesson tells us that any nonzero number raised to a negative power equals its reciprocal raised to the opposite or positive power.

$$x^{-n} = \frac{1}{x^n}$$

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Note also: any nonzero number raised to a positive power equals its reciprocal raised to the opposite or negative power.

$$x^m = \frac{1}{x^{-m}}$$

$$5^3 = \frac{1}{5^{-3}}$$

Power Rule

The "power rule" tells us that to raise a power to a power, just multiply the exponents. Here you see that 5^2 raised to the 3rd power is equal to 5^6.

$$(x^m)^n = x^{mn}$$

$$(5^2)^3 = 5^{2\times 3} = 5^6$$

[This also makes sense. Note: $(5^2)^3 = 5^2 \cdot 5^2 \cdot 5^2 = (5 \cdot 5) \cdot (5 \cdot 5) \cdot (5 \cdot 5) = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^6$]

Study these examples to see a few common errors that students make when working with exponents.

1. **The exponent next to a number applies only to that number unless there are parentheses indicating otherwise.**

$$-4^2 \neq (-4)^2$$
$$-4^2 = -4 \cdot 4 = -16$$
$$(-4)^2 = (-4)(-4) = 16$$

$$2m^2 \neq (2m)^2$$
$$2m^2 = 2 \cdot m \cdot m$$
$$(2m)^2 = (2m)(2m) = 4m^2$$

2. **The product rule** $(x^m x^n = x^{m+n})$ **only applies to expressions with the same base.**

$$4^2 \cdot 2^3 \neq 8^{2+3}$$
$$4^2 \cdot 2^3 = 4 \cdot 4 \cdot 2 \cdot 2 \cdot 2 = 128$$
$$8^{2+3} = 8^5 = 32{,}768$$

3. **The product rule** $(x^m x^n = x^{m+n})$ **applies to the product, not the sum, of two numbers.**

$$2^2 + 2^3 \neq 2^{2+3} \quad \text{(Note: They are not like terms.)}$$
$$2^2 + 2^3 = 2 \cdot 2 + 2 \cdot 2 \cdot 2 = 4 + 8 = 12$$
$$2^{2+3} = 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

<u>Calculators</u>

When calculating an exponent using a calculator, look for the following buttons and type in the sequence that follows. (Note: the "=" button might have enter written on it instead.)

$3^4 \Rightarrow \boxed{3}\boxed{y^x}\boxed{4}\boxed{=}$

$3^4 \Rightarrow \boxed{3}\boxed{\wedge}\boxed{4}\boxed{=}$

Some notes:

4. **Some problems are approached more easily if you think of them in pieces.**

Example: $\dfrac{15x^4y^5z^3}{25x^9y^2z^3}$

Although we wouldn’t show our work this way, let’t think of each part of the expression separately, simplify it, and then put them all together.

$$\frac{15}{25}=\frac{3}{5} \qquad \frac{x^4}{x^9}=x^{-5} \text{ or } \frac{1}{x^5} \qquad \frac{y^5}{y^2}=y^3 \qquad \frac{z^3}{z^3}=1$$

So, $\dfrac{15x^4y^5z^3}{25x^9y^2z^3}=\dfrac{3}{5}\cdot\dfrac{1}{x^5}\cdot y^3\cdot 1=\dfrac{3y^3}{5x^5}$

5. **Often times a simplified problem is considered one without negative exponents or parentheses.**